Chains and cycles.

Let $(\gamma_j)_{j=1}^{n}$ be a finite collection of arcs (piecewise differentiable).

$(\alpha_j)_{j=1}^{n}$ - a finite collection of complex numbers.

$\gamma = \alpha_1\gamma_1 + \ldots + \alpha_n\gamma_n$ - a formal sum.

$$\oint_{\gamma} f(z)dz \overset{def}{:=} \alpha_1 \oint_{\gamma_1} f(z)dz + \ldots + \alpha_n \oint_{\gamma_n} f(z)dz$$

Def. Two chains are equivalent if they can be obtained one from another by a sequence of the following operations:

1) Permutation of two arcs. $\alpha_1\gamma_1 + \alpha_2\gamma_2 = \alpha_2\gamma_2 + \alpha_1\gamma_1$.
2) Subdivision of an arc
3) Fusion of two arcs with the same coefficients and matching endpoints to form a single arc.
4) Reparametrization of an arc.
5) Cancellation of opposite arcs: $\alpha\gamma_1 + \beta(-\gamma_1) = (\alpha-\beta)\gamma$.

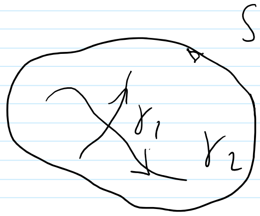
The value of $\oint_{\gamma} f(z)dz$ does not change after these operations.

Each chain can be written as $\gamma = \alpha_1\gamma_1 + \ldots + \alpha_n\gamma_n$ where all γ_n are different.

There is also a null-chain $\gamma = 0$.

Remark. We also denote by γ the union $\bigcup_{j=1}^{n}\gamma_j$.

Strictly speaking, we should use different notations!



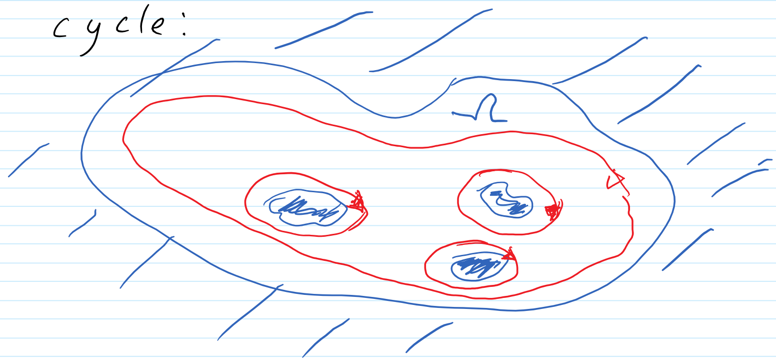
$i\gamma_1 - (2+i)\gamma_2 + 3i\gamma_3$.

Def. A chain is called a cycle if all γ_i can be chosen to be closed.

$$\gamma = \alpha_1\gamma_1 + \dots + \alpha_n\gamma_n, \text{ where } \gamma_1,\dots,\gamma_n \text{ - closed.}$$

Linear combination of two cycles is again a cycle.

My favorite cycle:



Oriented boundary.

Let $\gamma = \gamma_1 + \dots + \gamma_n$ be a cycle, each of the γ_i is a simple closed arc, $\gamma_j \cap \gamma_k = \emptyset$. We say that γ is an oriented boundary of region Ω (or γ bounds Ω) if

1) $\forall z \in \Omega$, $n(\gamma, z) = 1$

2) $\forall z \in (\Omega \cup \gamma)^c$, $n(\gamma, z) = 0$